Evolutionary Damping Constant

Evrimsel Sönüm Sabiti

Symbol: Ψ (Capital Psi)

[İsmail Kara][1]

[1] *[Körfez Atatürk Anadolu Lisesi] — [ismailkarailetisim@gmail.com]*

ORCID: [0009-0009-5651-0193]

Abstract

This study defines a new mathematical constant that brings face to face, within a generalized continued-fraction framework, the **Fibonacci sequence** — nature's symbol of order — and the sequence of **prime numbers** — mathematics' representative of chaos and unpredictability. Named the **Evolutionary Damping Constant** (*Ψ*), this constant is defined as the limiting value of an infinite generalized continued fraction whose denominators are Fibonacci numbers and whose numerators are prime numbers. Following the numerical and asymptotic corrections made in this English revision (see the Remark in Section 2.2), numerical computation places the constant at $\Psi \approx 0.896053$. A proof framework based on the Tietze–Legendre irrationality criterion is presented, showing that the exponential growth rate of the Fibonacci sequence permanently overtakes the logarithmic growth rate of the primes from a specific index onward, and it is argued that this crossover renders the constant irrational. The study concludes with a speculative discussion of possible applications in population genetics (mutation thresholds) and post-quantum cryptography.

Keywords: Evolutionary Damping Constant, generalized continued fraction, Fibonacci sequence, prime numbers, irrationality proof, Tietze–Legendre criterion, population dynamics

1. Introduction and Motivation

Mathematical constants reflect the deep connections between number theory and the applied sciences. Irrational constants such as the Golden Ratio ($\varphi \approx 1.618\ldots$) and π have become symbols of universal structures arising across different disciplines. With a similar motivation, this study proposes a new constant that unites two different, yet opposing, mathematical structures found in nature within a single formula.

In nature, population growth, the arrangement of plant shoots, and spiral structures can be explained by the Fibonacci sequence ($F_n = 1, 1, 2, 3, 5, 8, 13, \ldots$). This sequence exhibits growth close to exponential speed. Prime numbers ($p_n = 2, 3, 5, 7, 11, 13, \ldots$), on the other hand, grow logarithmically according to the Prime Number Theorem, in the form $p_n \sim n \ln n$, and their distribution remains connected to one of the deepest still-unsolved open problems in mathematics — the Riemann Hypothesis.

Comparing these two structures within a generalized continued fraction offers an original framework both for investigating the existence of a theoretical equilibrium point and for answering the question of 'when and how the chaos of the primes is suppressed by the order of the Fibonacci sequence.'

2. Mathematical Definition

2.1. Generalized Continued Fraction Representation

The Evolutionary Damping Constant Ψ is defined by the following generalized continued fraction:

$$\Psi = p_1 / (f_1 + p_2 / (f_2 + p_3 / (f_3 + p_4 / (f_4 + \cdots))))$$

where:

- **p_n**: the *n*-th prime number (2, 3, 5, 7, 11, 13, 17, 19, 23, …)

- **f_n**: the *n*-th Fibonacci number (1, 1, 2, 3, 5, 8, 13, 21, 34, …)

Unlike standard continued fractions (in which all numerators equal 1), here both the numerator and the denominator at each level are determined by independent sequences. Such structures are called **generalized continued fractions** in the literature.

2.2. Partial Convergents

To approximate the value of the infinite fraction, the partial convergents are computed:

C_1	$p_1 / f_1 = 2 / 1 = 2.0000$
C_2	$p_1 / (f_1 + p_2/f_2) = 2 / (1 + 3/1) = 0.5000$
C_3	≈ 1.0769
C_4	≈ 0.8358
C_5	≈ 0.9079 (oscillation begins narrowing)
C_∞	Ψ ≈ 0.896053…

The convergents oscillate above and below the true limiting value, drawing progressively closer to it. The narrowing of this oscillation — and its convergence to a limit — shows that the fraction converges.

Remark (Corrected Numerical Values). The convergents and the limiting value have been recomputed using exact rational arithmetic (Python's Fraction type, verified stable from depth 40 through depth 100 to 40 decimal digits). This computation confirms $C_3 \approx 1.0769$ as in the original draft, but corrects C_4 to 0.8358 (originally 0.8350), C_5 to 0.9079 (originally 0.9700), and — most importantly — the limiting value to **$\Psi \approx$ 0.8960525524489565…** (originally reported as 0.9423…). This correction does not affect the logical structure of the irrationality argument in Section 3, which depends only on the eventual dominance of the Fibonacci denominators, not on the precise decimal value of Ψ.

3. Proof of Irrationality

3.1. Proof Strategy

For simple continued fractions in which all numerators equal 1, direct irrationality results are relatively easy to obtain. For generalized fractions in which both the numerator and the denominator vary, however, different tools are required. This study applies the **Tietze–Legendre Irrationality Criterion**.

3.2. The Tietze–Legendre Criterion

Theorem: Let (*a_n*) and (*b_n*) be two sequences of positive integers. If, from some index N onward, the condition $a\_n \le b\_n$ holds permanently for all $n \ge N$, then the tail of the generalized continued fraction beyond step N is irrational (Rockett & Szüsz, 1992; Lorentzen & Waadeland, 2008).

In this study, the denominators *(f_n)* and numerators *(p_n)* play the roles of *b_n* and *a_n* respectively; the index from which the condition $f\_n \ge p\_n$ becomes permanent will be determined for application of the criterion.

3.3. Steps of the Proof

Step 1	**Asymptotic Growth Rates** The Fibonacci sequence grows at exponential speed, by Binet's Formula: f_n $\varphi^n/\sqrt{5}$ ($\varphi \approx 1.618…$). By the Prime Number Theorem, the n-th prime grows at logarithmic speed: p_n n ln n. Exponential growth permanently overtakes logarithmic growth in the long run.
Step 2	**Determining the Crossover Point** At the first several indices, the primes exceed the Fibonacci numbers: - $n = 1$: $p_1 = 2$, $f_1 = 1 \rightarrow p > f$ - $n = 6$: $p_6 = 13$, $f_6 = 8 \rightarrow p > f$ - $n = 7$: $p_7 = 17$, $f_7 = 13 \rightarrow p > f$ **Critical Point — n = 8:**

	- $n = 8$: $p_8 = 19$, $f_8 = 21 \rightarrow$ **f > p (first crossover)** - $n = 9$: $p_9 = 23$, $f_9 = 34 \rightarrow f > p$ - $n = 10$: $p_{10} = 29$, $f_{10} = 55 \rightarrow f > p$ - $n = 20$: $p_{20} = 71$, $f_{20} = 6765 \rightarrow f >> p$ (the gap has become a chasm)
Step 3	**Tail Irrationality and Permanence** From index $n = 8$ onward, the condition $f_n > p_n$ holds permanently for all $n \geq 8$, owing to the asymptotic difference between the exponential growth of the Fibonacci sequence and the logarithmic growth of the primes. This permanence has been verified computationally through $n = 300$ with no reversal, and is guaranteed indefinitely thereafter since exponential growth dominates $n \ln n$ growth for all sufficiently large n. By the Tietze–Legendre Criterion, the infinite tail of the fraction beyond step 8 is irrational.
Step 4	**The Result for the Full Constant** The first 7 partial layers of the fraction involve only rational operations (addition and division) applied to this irrational tail; as a mathematical fact, a rational number combined with an irrational number by addition or by division (with a nonzero rational numerator or divisor) always yields an irrational result. Applying this seven times as the construction unwinds outward from the irrational tail, the whole of Ψ is therefore irrational. ■

THEOREM: Ψ is an irrational number. (Ψ ≈ 0.896053…)

4. Theoretical Interpretation and Meaning

4.1. The 'Eighth Generation' Break

The threshold **N = 8** that emerges throughout the proof is not merely a technical crossover point. Interpreted within a biological metaphor: a virus (the primes) afflicting a population can outpace the population's natural rate of reproduction (Fibonacci) for the first 7 generations. From the 8th generation onward, however, nature's exponential growth power permanently suppresses the virus's logarithmic spread, and the system settles into an equilibrium point — Ψ.

It is significant that this breaking point is not coincidental, but arises from an asymptotic necessity between the growth rates of the two sequences.

4.2. Connection to the Riemann Hypothesis

The distribution of the primes is connected to the Riemann Hypothesis, the most important unsolved problem in mathematics. The constant Ψ defined in this study represents the boundary value of a confrontation in which an unsolved chaos (the primes) becomes trapped within a well-understood order (Fibonacci). In this respect, Ψ carries a symbolic value that embodies the tension between two deep mathematical structures, beyond simply being a numerical constant.

5. Possible Application Areas

The interpretations in this section are explicitly speculative and are offered as conceptual analogies rather than established scientific models.

5.1. Population Genetics and the Mutation Threshold

Considering an ecosystem in which individuals reproduce at Fibonacci-indexed ratios and random mutations (or pathogen attacks) arise at prime-indexed generations, the constant Ψ can be interpreted as an **evolutionary resilience threshold**:

- A pathogen whose spread rate exceeds Ψ → population collapse

- A pathogen whose spread rate stays below Ψ → immunity and adaptation

While this model is speculative, it opens the door to an analogical interpretation connected to the concept of the *basic reproduction number* (R_0) in the mathematical epidemiology literature.

5.2. Cryptography

Today's asymmetric encryption systems rely on the computational difficulty of factoring large primes. While advances in quantum computing may reduce this difficulty, the prime–Fibonacci pattern in the definition of Ψ may provide raw material for a new, hard-to-break **post-quantum encryption** key structure. This remains an open question at the research level.

6. Limitations and Open Questions

1. **Numerical precision:** The corrected value $\Psi \approx 0.896053\ldots$ has been obtained via partial-convergent computation with exact rational arithmetic, stable to 40 decimal digits. Analytic determination of rigorous upper and lower bounds remains to be established.

2. **Is it transcendental?** Whether Ψ is merely irrational or also a transcendental number remains undetermined.

3. **Rate of convergence:** A theoretical characterization of the rate at which the partial convergents approach Ψ is important for potential applications.

4. **A general family of fractions:** Besides the p_n / f_n fraction, investigating the f_n / p_n arrangement, and the roles of primes and Fibonacci numbers in other fraction structures, may lead to a broader family of constants.

7. Conclusion

This study has defined a new mathematical constant that unites the chaotic growth of the primes with the orderly growth of the Fibonacci sequence within a generalized continued-fraction framework. The proof framework based on the Tietze–Legendre criterion shows that, from index n = 8 onward, the Fibonacci sequence permanently overtakes the primes, and that this condition renders Ψ irrational.

From a mathematical standpoint, the idea that a sequence directly connected to the still-unsolved Riemann Hypothesis can be 'trapped' by a much better-understood sequence remains an intriguing new research question. Determining the transcendence of Ψ and refining its precise numerical value remain priority goals for future work.

References

Davenport, H. (2008). *The Higher Arithmetic* (8th ed.). Cambridge University Press.

Euler, L. (1748). *Introductio in Analysin Infinitorum*. Lausanne.

Hardy, G. H., & Wright, E. M. (2008). *An Introduction to the Theory of Numbers* (6th ed.). Oxford University Press.

Livio, M. (2002). *The Golden Ratio: The Story of Phi, the World's Most Astonishing Number*. Broadway Books.

Lorentzen, L., & Waadeland, H. (2008). *Continued Fractions: Vol. 1 — Convergence Theory*. Atlantis Press.

Riemann, B. (1859). Über die Anzahl der Primzahlen unter einer gegebenen Grösse. *Monatsberichte der Berliner Akademie*.

Rockett, A. M., & Szüsz, P. (1992). *Continued Fractions*. World Scientific.

İsmail Kara | ismailkarailetisim@gmail.com | ORCID: 0009-0009-5651-0193
ismailkara.com.tr